Reliability Improvement via Degradation Experiments

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Abstract

Assuming that the product's degradation paths satisfy a non-linear diffusion process, this paper proposes a systematic approach to improve the mean lifetime (MTTF) of highly reliable products. First, an intuitively appealing identification rule is proposed. Next, under the constraint of a minimum probability of correct decision and a maximum probability of incorrect decision of the proposed identification rule, the optimum test plan (including the determination of inspection frequency, sample size, and termination time for each run) can be obtained by minimizing the total experimental cost. An example is provided to illustrate the proposed method.

1 Introduction

Due to strong pressure for marketing, continuous improvement of a product's reliability has become necessary for manufacturers to compete with others. Hence, the following fundamental work is essential: improving the product's reliability if it does not match customers' requirement. Statistical design of experiments (DOE) is always adopted to identify the factors that are influential to the product's reliability. Condra (1993) gave some examples of using DOE to improve reliability in the area of electronics.

For a highly-reliable product, it is difficult or nearly impossible to assess the product's lifetime by using traditional life tests or accelerated life tests. Rather, if there exist product characteristics whose degradation over time can be related to reliability, then collect "degradation data" of those characteristics can provide information about the product's reliability. Meeker & Escobar (1998, Chapters 13 and 21) gave an updated literature survey on the approaches of assessing reliability information via degradation data.

In conducting a degradation experiment, several decision variables such as inspection frequency, sample size, and termination time for each run are influential to the correct identification of significant factors and the experimental cost. Recently, Tseng, *et al.* (1995) and Yu & Tseng (2002) used a transformed degradation model to demonstrate how to determine the optimal settings of these decision variables so that the significant factors can be picked up successfully. In the above approach, however, the lifetime inference is very sensitive the precision of estimated parameters in their degradation models. To overcome this difficulty, motivating from an LED degradation data, this article uses a stochastic diffusion model to describe the product's degradation path. In the following, we first describe the experimental layout of a degradation experiment. Then we propose a general non-linear degradation model and we use an optimization model to solve the optimal settings of a degradation experiment.

2 Problem Formulation

2.1 The experimental layout

Assume that an orthogonal array $OA(r, (2^m)$ with degradation model is conducted as follows:

Run	Factors $F_1 \ \cdots \ F_h \ \cdots \ F_m$	Degradation paths
1	$c_{11} \ \cdots \ c_{1h} \ \cdots c_{1m}$	$\{L_{1j}(t_k)\}_{k=1,j=1}^{l,\quad n}$
$\vdots$	$\vdots$	$\vdots$
i	$c_{i1} \ \cdots \ c_{ih} \ \cdots c_{im}$	$\{L_{ij}(t_k)\}_{k=1,j=1}^{l,\quad n}$
$\vdots$	$\vdots$	$\vdots$
r	$c_{r1} \ \cdots \ c_{rh} \ \cdots c_{rm}$	$\{L_{rj}(t_k)\}_{k=1,j=1}^{l,\quad n}$

where the design points, r, is a multiple of 4, all the factors have two levels, and the number of factors is $m \leq r-1$.

Assumptions:

1. For each run, there are m factors, say $F_1, F_2, \ldots, F_m$, that may affect the product's reliability. Assume that there exist no interactions among these factors.
2. Two levels of each factor are denoted by $(-,+)$ and referred to as the (low, high) settings. More specifically, For each $1 \leq i \leq r$, and $1 \leq h \leq m$, we define

$$c_{ih} = \begin{cases} -1 & \text{if } F_h \text{ is evaluated at low level} \\ 1 & \text{if } F_h \text{ is evaluated at high level.} \end{cases}$$

3. At each run, n devices are randomly selected for testing.
4. At each run, the measurements are made per f units of time (e.g., f hours or f days) until the time $t_l = f * l$, where l is a positive integer.
5. The degradation model:
   Let $L_{ij}(t)$ denote the degradation path of the j^{th} device under the i^{th} run at time t. Motivating by Tseng & Peng (2004), we assume that there exists a suitable function $s_i(t)$ such that

$$L_{ij}(t) = M_i(t) + \epsilon_{ij}(t), t \geq 0,$$

   where

$$d\epsilon_{ij}(t) = s_i(t) dB(t).$$

   Note that $M_i(t)$ denote the mean degradation path under the i^{th} run and $B(t)$ be a standard Brownian motion (Wiener process).
6. The lifetime of the product (T_{ij}): T_{ij} is defined as the first time when $L_{ij}(t)$ falls below a critical level ω.

2.2 An identification rule

For $1 \leq h \leq m$, let E_h denote the main effect of factor F_h. Then

$$E_h = \frac{2}{r}\left(\sum_{i=1}^{r} c_{ih}\theta_i\right), \tag{1}$$

where θ_i denotes the mean-time-to-failure (MTTF) of the i^{th} run.

For pre-specified constants Δ_0 and Δ_1 ($\Delta_1 > \Delta_0 > 0$), if $| E_h | \geq \Delta_1$, then we say that F_h is practically significant; If $| E_h | \leq \Delta_0$, then we say that F_h is practically negligible.

Let $[\underline{E}_h, \bar{E}_h]$ be the corresponding $100(1-\gamma)\%$ confidence interval of E_h. Then an identification rule (R) is proposed as follows:

R: *For $1 \leq h \leq m$, F_h is identified as a statistically significant factor if $0 \notin [\underline{E}_h, \bar{E}_h]$,*

while F_h is identified as a statistically negligible factor if $0 \in [\underline{E}_h, \bar{E}_h]$.

3 The optimization problem

Suppose that there are $q(\leq m)$ factors of practical significance. In this case, we will assume without loss of generality that $F_1, F_2, \ldots, F_q$ are the significant factors. Let **Q** denote the set of indices of all significant factors, i.e., $\mathrm{Q} = \{1, 2, 3, \ldots, q\}$. The goal is to select the set of all practically significant factors, while controlling the maximum probability of false positives for insignificant effects as small as we like.

Define

$$\Omega = \{\vec{E} = (E_1, E_2, \ldots, E_m) | \; | E_i | < \infty, 1 \leq i \leq m\}.$$

The following two subsets (Ω_0 and Ω_Q) of Ω are critical to the decision maker. Let

$$\Omega_Q = \{\vec{E} = (E_1, E_2, \ldots, E_m) | \min_{1 \leq h \leq q} | E_h | \geq \Delta_1\}$$

and

$$\Omega_0 = \{\vec{E} = (E_1, E_2, \ldots, E_m) | \max_{1 \leq h \leq m} | E_h | \leq \Delta_0\}.$$

Note that Ω_0 denotes the set of all practically negligible effects and Ω_Q denotes the set of all practically significant effects. We say that Rule **R** gives a correct decision (CD) for $\vec{E} = (E_1, E_2, \ldots, E_m) \in \Omega_Q$, if F_h is identified as a statistically significant factor for all $h \in \mathrm{Q}$. Similarly, Rule **R** gives an incorrect decision (ICD) for $\vec{E} = (E_1, E_2, \ldots, E_m) \in \Omega_0$, if F_h is identified as a statistically significant factor for some h, $1 \leq h \leq m$.

Let $P_{\vec{E}}(CD \mid R)$ and $P_{\vec{E}}(ICD \mid R)$ denote the probabilities that Rule **R** gives correct decisions and incorrect decisions for $\vec{E}$, respectively. To enhance the quality of our decision, we usually impose the following conditions:

$$\inf_{\vec{E} \in \Omega_Q} P_{\vec{E}}(CD \mid R) \geq 1 - \beta, \tag{2}$$

and

$$\sup_{\vec{E} \in \Omega_0} P_{\vec{E}}(ICD \mid R) \leq \alpha, \tag{3}$$

where $1-\beta$ is the power of the procedure, the minimum probability of correctly identifying effects of practical significance, and α is the significance level, the maximum probability of false positives for negligible effects. Note that, α and β are pre-determined values given by the decision maker.

This may lead to several combinations of decision variables (f, l, n) that satisfy Equations (2) and (3). However, the decision variables (f, l, n) will affect the experimental cost. Thus, a trade-off is needed. Let $\mathrm{TC}(f, l, n)$ denote the total cost of conducting the experiment. Then a typical decision problem can be formulated as follows:

Minimize

$$\mathrm{TC}(f, l, n) \tag{4}$$

Subject to

$$\inf_{\vec{E}\in\Omega_q} P_{\vec{E}}(CD \mid R) \geq 1-\beta$$

$$\sup_{\vec{E}\in\Omega_0} P_{\vec{E}}(ICD \mid R) \leq \alpha$$

where f, l, $n \in \mathbf{N}$=$\{1, 2, 3, \ldots\}$.

4 Optimum Test Plan

The framework of accomplishing the optimization model consists of the following four parts:

1. Characterize the cost function $\mathrm{TC}(f, l, n)$.
2. Estimate E_h.
3. Compute the confidence interval $[\underline{E}_h, \bar{E}_h]$.
4. Compute $\inf_{\vec{E}\in\Omega_s} P_{\vec{E}}(CD \mid R)$ and $\sup_{\vec{E}\in\Omega_0} P_{\vec{E}}(ICD \mid R)$.

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